

SOP 2

Recommended Standard Operating Procedure for Applying Air Buoyancy Corrections

1. Introduction

1.1. Purpose

If uncorrected, the effect of air buoyancy on the objects being compared is frequently the largest source of error in mass measurement. This SOP provides the equations to be used to correct for the buoyant effect of air. The significance of the air buoyancy correction depends upon the accuracy required for the measurement, the magnitude of the air buoyancy correction relative to other sources of error in the overall measurement process, and the precision of the mass measurement. An air buoyancy correction should be made in all high accuracy mass determinations. The gravimetric volume procedure uses a high accuracy mass determination with the corresponding buoyancy corrections. The Appendix to this SOP provides a brief theoretical discussion of this subject.

1.2. Prerequisites

1.2.1. Verify that (true) mass values or corrections are available for the standards used in the measurement process and that they have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST.

1.2.2. Verify that the thermometer, barometer, and hygrometer used have been calibrated, and that they have demonstrated metrological traceability to the international system of units (SI), which may be to the SI through a National Metrology Institute such as NIST, and are in good operating condition as verified by periodic tests or cross-checks with other standards.

2. Methodology

2.1. Scope, Precision, Accuracy

This procedure is applicable to all weighings using a comparison of mass standards. The precision will depend upon the accuracy of the thermometer, barometer, and hygrometer used to determine the air density. When the calculations for the air density and air buoyancy corrections are made, a sufficient number of decimal places must be carried so the error due to the rounding of numbers is negligible relative to the error in the measurement process. Typically, carrying six to eight decimal places is sufficient, but depends upon the precision of the measurement.

2.2. Apparatus/Equipment Required¹

- 2.2.1. Calibrated barometer with sufficiently small resolution, stability, and uncertainty (See SOP 2, e.g., accurate to ± 66.5 Pa (0.5 mm Hg)) to determine barometric pressure.
- 2.2.2. Calibrated thermometer with sufficiently small resolution, stability, and uncertainty (see SOP 2, e.g., accurate to ± 0.10 °C) to determine air temperature.
- 2.2.3. Calibrated hygrometer with sufficiently small resolution, stability, and uncertainty (see SOP 2, e.g., accurate to ± 10 percent) to determine relative humidity.

2.3. Estimating the Magnitude of the Air Buoyancy Correction

- 2.3.1. Estimate the magnitude of the air buoyancy correct, *MABC*, using the following formula:

$$MABC = (\rho_a - \rho_n)(V_x - V_s)$$

- 2.3.2. The equation may also be represented as follows:

$$MABC = m_o (\rho_a - \rho_n) \left(\frac{1}{\rho_x} - \frac{1}{\rho_s} \right)$$

Table 1. Variables for *MABC* equation.

Variable	Description of Variable
ρ_a	air density at the time of the measurement in mg/cm^3
ρ_n	density of "normal" air; i.e., $1.2 \text{ mg}/\text{cm}^3$
m_o	nominal mass (in grams)
V_x	volume of the unknown weight, <i>X</i> in cm^3
V_s	volume of the reference standard, <i>S</i> in cm^3
ρ_s	density of reference standard, <i>S</i> in g/cm^3
ρ_x	density of unknown weight, <i>X</i> in g/cm^3

The relative magnitude of the correction can be compared to the expanded measurement uncertainty to determine the importance of the air buoyancy correction for a particular measurement. In some mass calibration procedures, when the calculated value is sufficiently small compared to applicable tolerances, the value

¹ The barometer, thermometer, and hygrometer are used to determine the air density at the time of the measurement. The air density is used to make an air buoyancy correction. The accuracies specified are recommended for high precision calibration. Less accurate equipment can be used with only a small degradation in the overall accuracy of the measurement.

may be treated as an uncorrected systematic error and incorporated into the uncertainty calculations.

2.4. Procedure

- 2.4.1. Record the temperature, pressure, and relative humidity at the start and at the end of the measurement process as near the location of the measurement as necessary and practical. If these parameters change significantly during the measurement process, it may be necessary to wait for more stable operating conditions or to use average values to compute the air density. Use of the average environmental values may influence the uncertainty of the measurement result and must be evaluated for significance.
- 2.4.2. Determine the air density using the equation given in Section 8 of the Appendix to this SOP.

3. Calculations

- 3.1. Calculate the mass, M_x , of the unknown weight, X , using the following equation, where d represents the “difference” obtained with buoyancy corrections applied to the sensitivity weight.

$$M_x = \frac{M_s \left(1 - \frac{\rho_a}{\rho_s} \right) + d}{\left(1 - \frac{\rho_a}{\rho_x} \right)}$$

If tare weights were carried with X and/or S , use the following equation:

$$M_x = \frac{M_s \left(1 - \frac{\rho_a}{\rho_s} \right) + d + M_{t_s} \left(1 - \frac{\rho_a}{\rho_{t_s}} \right) - M_{t_x} \left(1 - \frac{\rho_a}{\rho_{t_x}} \right)}{\left(1 - \frac{\rho_a}{\rho_x} \right)}$$

Table 2. Variables not previously defined.

Variable	Description of Variable
d	measured difference between X and the reference standard, S, using one of the weighing designs given in other SOPs
M_s	[true] mass of the reference standard
M_{t_s}	[true] mass of the tare weights carried with S
M_{t_x}	[true] mass of the tare weights carried with X
ρ_s	density of the reference standard, S
ρ_x	density of the unknown standard, X
ρ_{t_s}	density of the tare weights carried with S
ρ_{t_x}	density of the tare weights carried with X

3.2. If reporting the conventional mass², CM_x , compute it using the following.

$$CM_x = \frac{M_x \left(1 - \frac{0.0012}{\rho_x} \right)}{\left(1 - \frac{0.0012}{8.0} \right)}$$

3.3. If reporting the apparent mass, AM_x , versus brass, compute it using the following.

$$AM_{x \text{ vs brass}} = \frac{M_x \left(1 - \frac{0.0012}{\rho_x} \right)}{\left(1 - \frac{0.0012}{8.3909} \right)}$$

3.4. The conventional and apparent mass values are related by the following:

$$CM_x = \frac{AM_{x \text{ vs brass}} \left(1 - \frac{0.0012}{8.3909} \right)}{\left(1 - \frac{0.0012}{8.0} \right)}$$

² Conventional Mass: “The conventional value of the result of weighing a body in air is equal to the mass of a standard, of conventionally chosen density, at a conventionally chosen temperature, which balances this body at this reference temperature in air of conventionally chosen density.” The conventions are: reference density 8.0 g/cm³; reference temperature 20 °C; *normal* air density 0.0012 g/cm³. Conventional mass was formerly called “Apparent Mass versus 8.0 g/cm³” in the United States and is not recommended. *See OIML D28 (2004)*.

4. Assignment of Uncertainty

The uncertainty in determining the air buoyancy correction is usually negligible relative to the precision of the measurement process itself. Consequently, the uncertainty for the measurement is based upon the uncertainty for the measurement process used and the uncertainty associated with the environmental measurement standards. The uncertainty in the CIPM³ 2007 air density equation is 0.000 026 4 mg/cm³ (or 0.002 2 % of normal air density.)

Table 3. Tolerances for measurements related to air density estimation.

Variable	Uncertainty of air density values in % of air density		
	± 0.1 % of air density	± 1.0 % of air density	Recommended (Section 2.2)
Air pressure (Pa)	± 101	± 1010	± 66.5
Air pressure (mm Hg)	± 0.76	± 7.6	± 0.5
Air temperature (°C)	± 0.29	± 2.9	± 0.1
Relative Humidity (%)	± 11.3	± 10

³ CIPM References: A Picard, R.S. Davis, M. Gläser, K. Fujii, Metrologia 45:149-155 (2008).

Appendix A

Based on “The Basic Theory of Air Buoyancy Corrections”

by Richard S. Davis⁴

1. Introduction

In performing measurements of mass, the balance or scale used acts as a force transducer. The force produced by an object to be weighed in air has two components: one proportional to the mass of the object, and the other proportional to its volume. The latter component, or buoyant force, may under some circumstances be large enough to require correction. The following shows under what circumstances buoyancy corrections are required as well as how they are made.

2. Scope

The method for applying buoyancy corrections presented below applies to mass measurements made in air. The density of air is computed from auxiliary measurements of temperature, pressure and relative humidity after which the buoyancy corrections are calculated directly from the Principle of Archimedes. The following weighing situations are considered.

2.1. Two-Pan Balance

2.2. Single-Pan Balance

2.2.1. With Built-In Weights

2.2.2. With Electronic Control

3. Summary of Method

In general, buoyancy corrections are applied to mass measurements by calculating the difference in volume between the unknown weight and the standard, multiplying this volume difference by the density of air at the balance or scale, and adding the product to the mass of the standard. The density of air is computed from an equation of state using measured values for the temperature, pressure and relative humidity of the air.

4. Significance and Use

Buoyancy corrections generally must be applied when determining the mass of an unknown object to high accuracy. The corrections may become important even at modest

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accuracies if the unknown object whose mass is to be determined has a density that differs widely from that of the standards (weighing of water, for example). Many mass standards are calibrated in terms of a so-called "apparent mass" [conventional mass] scale (See Chapter 7.3, Handbook 145¹). Use of this scale does not indiscriminately eliminate the need for buoyancy corrections as is sometimes assumed.

5. Terminology

5.1. Weighing by Substitution

Substitution weighing is the procedure by which one determines the assembly of standard weights that will produce nearly the same reading on a one-pan balance as does the unknown object. The balance thus serves as a comparator. A two-pan balance may be used in this mode if one of the pans contains a counterpoise and the standards and unknown are substituted on the second pan. (See SOP No. 3.)

5.2. Mass

The term "mass" is always used in the strict Newtonian sense as a property intrinsic to matter. Mass is the proportionality constant between a force on a material object and its resulting acceleration. This property is sometimes referred to as "true mass", "vacuum mass", or "mass in vacuum" to distinguish it from conventional [apparent] mass.

5.3. Conventional [Apparent] Mass⁵

The mass of material of a specified density that would exactly balance the unknown object if the weighing were carried out at a temperature of 20 °C in air of density 0.0012 g/cm³. The mass, M_N , of an object, N , is related to its apparent mass $M_{N,A}$ by the equation:

$$M_{N,A} = \frac{M_N \left(1 - \frac{0.0012}{\rho_N} \right)}{\left(1 - \frac{0.0012}{\rho_B} \right)}$$

⁵Pontius, P. E., Mass and Mass Values, NBS Monograph 122, 1974, pp 12-23, 26-33 described the concept of apparent mass. The term conventional mass is described by OIML D28, the Conventional Value of Mass in Air.

Table A-1. Variables for conventional (apparent) mass equation.

Variable	Description
ρ_N	density of the object N at 20 °C in g/cm ³
ρ_B	density of the conventional (apparent) mass scale at 20 °C in g/cm ³

There are at present two apparent mass scales in wide use. The older is based on $\rho_B = 8.4000 \text{ g/cm}^3$ at 0 °C with a coefficient of volumetric expansion of $0.000054 / ^\circ\text{C}$ and the more recent (Conventional Mass) specifies $\rho_B = 8.0000 \text{ g/cm}^3$ at 20 °C. The quantity $M_{N,A}$ is a function of the particular conventional or apparent mass scale, which has been used in its calculation. OIML D28 only recognizes Conventional Mass.

5.4. Sensitivity

The response of a balance under load to an additional small weight:

$$\text{sensitivity} = \frac{M_{sw} - \rho_a V_{sw}}{\Delta R} = \frac{M_{sw} \left(1 - \frac{\rho_a}{\rho_{sw}} \right)}{\Delta R}$$

Table A-2. Variables for sensitivity equation.

Variable	Description
<i>sensitivity</i>	balance sensitivity (mass per division)
M_{sw}	mass of the small, additional weight
ρ_a	density of the air
ρ_{sw}	density of the small, additional weight
V_{sw}	volume of the small, additional weight
ΔR	change in balance reading due to the addition of the small weight, balance deflection

6. Apparatus

In order to ascertain the density of air at the balance, the following measuring instruments are necessary: thermometer, barometer, and hygrometer. Ideally, these instruments should be placed in or next to the balance case (as near the measurement location as is practical). It may only be practical for the thermometer or temperature sensor to actually be placed inside the balance chamber. A calculator or computer will be extremely useful for this procedure.

7. Procedure

Weigh the unknown object as directed by the balance manufacturer or in accordance with accepted procedure. Record the temperature, pressure and relative humidity of the air in the balance at the time of weighing. Do not correct the barometric pressure to sea level.

8. Calculation

8.1 Air density, Option A (Option B is preferred)

The density of air, in g/cm³, can be *approximated for lesser accuracy* from the following formula:⁶

$$\rho_a = \left\{ \frac{0.464\,60 [P - (0.003\,796\,0\,U\,e_s)]}{(273.15 + t)} \right\} \times 10^{-3} \quad (1)$$

Table A-3. Variables for air density equation.

Variable	Description
ρ_a	density of air, g/cm ³
P	barometric pressure, mm Hg
U	% relative humidity, entered as a whole number
t	temperature, °C
e_s	$1.314\,6 \times 10^9 \times e^{[-5\,315.56/(t + 273.15)]}$

Note: e_s can be written as follows in a spreadsheet and in some calculators:
1.3146E9*@EXP(-5315.56/(t+273.15))

Small errors (of order 0.01 %) in this equation occur for locations well above sea level or under conditions in which the concentration of carbon dioxide differs greatly from the global average. See the references for a more general formulation of the equation.

⁶Jones, F.E., "The Air Density Equation and the Transfer of the Mass Unit," Journal of Research, National Institute of Standards and Technology, Vol. 83, 1978, p. 419.

8.2. Air density, Option B – Preferred

The density of air should be calculated with the following formula.⁷

$$\rho_a = \frac{pM_a}{ZRT} \left(1 - x_v \left(1 - \frac{M_v}{M_a} \right) \right)$$

$$x_v = \left(h / 100 \right) f \frac{\rho_{sv}}{p}$$

$$Z = 1 - \frac{p}{T} \left[a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t) x_v + (c_0 + c_1 t) x_v^2 \right] + \frac{p^2}{T^2} (d + e x_v^2)$$

Table A-4. Variables for CIPM air density equation.

Variable	Description
M_a	molar mass of the air within laboratory 28.965 46 x 10 ⁻³ kg/mol
M_v	18.01528(17) x 10 ⁻³ kg mol ⁻¹
p	ambient barometric pressure in Pascal
T	ambient temperature in Kelvin
R	universal gas constant: 8.314 472 J mol ⁻¹ K ⁻¹
h	relative humidity in %
f	1.000 62 + (3.14 x 10 ⁻⁸) p + (5.6 x 10 ⁻⁷) t ²
t	ambient temperature in degrees Celsius
p_{sv}	1 Pascal x exp (AT ² + BT + C + D/T)
A	1.237 884 7 x 10 ⁻⁵ K ⁻²
B	-1.912 131 6 x 10 ⁻² K ⁻¹
C	33.937 110 47
D	-6.343 164 5 x 10 ³ K
a_0	1.581 23 x 10 ⁻⁶ K Pa ⁻¹
a_1	-2.933 1 x 10 ⁻⁸ Pa ⁻¹
a_2	1.104 3 x 10 ⁻¹⁰ K ⁻¹ Pa ⁻¹
b_0	5.707 x 10 ⁻⁶ K Pa ⁻¹
b_1	-2.051 x 10 ⁻⁸ Pa ⁻¹
c_0	1.989 8 x 10 ⁻⁴ K Pa ⁻¹
c_1	-2.376 x 10 ⁻⁶ Pa ⁻¹
d	1.83 x 10 ⁻¹¹ K ² Pa ⁻²
e	-0.765 x 10 ⁻⁸ K ² Pa ⁻²

⁷ CIPM References: Giacomo, P. Metrologia 18: 33-40 (1982), Davis, R.S., Metrologia 29: 67-70 (1992), A Picard, R.S. Davis, M. Gläser, K. Fujii, Metrologia 45:149-155 (2008).

Calculate the density of air at the balance during the weighing. Then determine the mass of the unknown, M_x , as follows:

8.3 If a two-pan balance is used:

$$M_x = M_s + \rho_a (V_x - V_s) + \text{sensitivity } \Delta M_{opt} \quad (3a)$$

or

$$M_x = \frac{M_s \left(1 - \frac{\rho_a}{\rho_s} \right) + \text{sensitivity } \Delta M_{opt}}{\left(1 - \frac{\rho_a}{\rho_x} \right)} \quad (3b)$$

or

$$M_x = \frac{M_s - \rho_a V_s + \text{sensitivity } \Delta M_{opt}}{\left(1 - \frac{\rho_a}{\rho_x} \right)} \quad (3c)$$

Table A-5. Variables not previously defined.

Variable	Description
M_x	mass of the unknown object
M_s	mass of the standard weights
ρ_s	density of the standard weights, $\frac{M_s}{V_s}$
ΔM_{opt}	off-balance indication read on the optical scale
V_x	volume of the unknown object
V_s	volume of the standard weights
ρ_x	density of the unknown object, $\frac{M_x}{V_x}$

Volumes and densities are, in general, a function of temperature. The thermal coefficients of volumetric expansion of the unknown object and the standard may be significant in very accurate work.⁸ The coefficient of volumetric expansion is

⁸In general, $V_t = V_{20} [1 + \beta(t - 20^\circ\text{C})]$ where t is the temperature of the weight, V_{20} is the volume at 20°C , and β is the cubical coefficient of expansion.

usually estimated as three times the linear coefficient of expansion of the weight material.

The error in M_x incurred by ignoring the buoyancy correction is $\rho_a (V_x - V_s)$. To estimate quickly whether such an error is of consequence in a particular measurement, (assume $\rho_a = 1.2 \times 10^{-3} \text{ g/cm}^3$).

If the mass and volumes of the standards have been adjusted to a conventional mass scale, then

$$M_x \approx \frac{CM_s \left(1 - \frac{\rho_a}{\rho_B} \right) + \text{sensitivity } \Delta M_{opt}}{\left(1 - \frac{\rho_a}{\rho_x} \right)} \quad (4)$$

Table A-6. Variables not previously defined.

Variable	Description
CM_s	conventional mass of the standard
The symbol \approx signifies an approximation	

- 8.4 If a single-pan balance with built-in weights is used, it is probable that the built-in weights have been adjusted on an apparent mass or conventional mass scale⁹. Determine which apparent mass scale has been used and calculate the mass of the unknown from the equation

$$M_x \approx \frac{M_D \left(1 - \frac{\rho_a}{\rho_B} \right) + \text{sensitivity } M_{opt}}{\left(1 - \frac{\rho_a}{\rho_x} \right)} \quad (5)$$

Table A-7. Variables not previously defined.

Variable	Description
M_D	mass indicated by dial or digital readings
M_{opt}	mass indicated on the optical scale when present

⁹Schoonover, R. M. and Jones, F. E., "Air Buoyancy in High-Accuracy Weighing on Analytical Balances," Anal. Chem., 53, 1981, p. 900.

If the balance has been used only as a comparator, that is, to compare the mass of the unknown object with that of some external standard, then:

$$M_x = M_s + \rho_a (V_x - V_s) + \text{sensitivity } \Delta M'_{opt}$$

Table A-8. Variable not previously defined.

Variable	Description
$\Delta M'_{opt}$	difference in optical scale reading between observations of the standard and the unknown

For some balances, operation requires that the user restore the balance to null by means of a manually controlled dial. The portion of the mass reading controlled by this dial should be treated, for purposes of buoyancy corrections, as an optical scale.

- 8.5 If a single-pan balance with full-range electronic control is used, the following should be noted. As part of its calibration, the electronic gain has been adjusted by means of a weight of known mass. For example, if the range of electronic control is 100 g, the electronics have been adjusted so that a 100 g standard weight produces an indication of precisely 100 g. This procedure effectively builds an apparent mass calibration into the balance. The reference density of the apparent mass scale is the density of the standard mass used for the calibration and the reference air density is the air density at the time of calibration.

The mass of an unknown object weighed on the balance is then

$$M_x = \frac{M_R \left(1 - \frac{\rho_a}{\rho_c} \right)}{\left(1 - \frac{\rho_a}{\rho_x} \right)} \quad (6)$$

Table A-9. Variables not previously defined.

Variable	Description
M_R	readout displayed on the balance
ρ_a	density of air at the time of balance calibration
ρ_c	density of the standard used to calibrate the balance (or ρ_B if the apparent [conventional] mass of the standard was used instead of the true mass)

If the balance includes both an electronic control system and built-in weights, the buoyancy considerations for the built-in weights are as described in section 8.2 and the considerations for the electronically determined mass are those given directly above.

8.6 Top-loading balances may be considered a form of single-pan balance and the appropriate procedure for buoyancy correction followed.

9. Precision

The contribution of the random error of the evaluation of air density to the precision of mass measurement may be estimated as follows:

For mechanical balances, or electronic balances used in weighing by substitution, the contribution is:

$$\delta\rho_a (V_x - V_s)$$

Table A-10. Variables for above equation.

Variable	Description
$\delta\rho_a$	random error of evaluation of ρ_a
V_s	volume of standards, if weighing by substitution $V_s = M_D / D_B$, if using the built-in weights on a single pan balance.
V_x	volume of object weighed

The quantity, $\delta\rho_a$ will have contributions from the measurements of temperature, pressure and relative humidity which are required for the calculation of ρ_a . Equation (1) may be used to estimate the effects of imprecision in measurements of P , t , and U . It is unrealistic to expect $\delta\rho_a/\rho_a$ ever to be less than 0.05 % even using the best techniques available.

10. Accuracy

Inattention to problems of buoyancy most often results in systematic errors. For a substitution weighing, for example, the buoyancy correction is of the order of $\rho_a(V_x - V_s)$. While this quantity may be significant to a measurement, day-to-day variation in ρ_a (usually no more than 3 %) may not be noticeable and hence need not be measured. For the most accurate work, not only must ρ_a be accurately determined, but the volumes of the unknown and standard may have to be measured to better than 0.05 % - the minimum systematic uncertainty attainable in the calculation of ρ_a .

If the standards have been calibrated in terms of conventional mass, complete neglect of buoyancy corrections will produce an error in the measured result M_x of order:

$$CM_s \left[\left(1.2 \times 10^{-3} - \rho_a \right) \left(\frac{1}{\rho_s} - \frac{1}{\rho_B} \right) + \left(\frac{\rho_B - \rho_x}{\rho_B} \right) \left(\frac{\rho_a}{\rho_x} \right) \right] \quad (7)$$

This error is often unacceptably large.

Use of equation (4), on the other hand, introduces only an error of approximately

$$CM_s \left(1.2 \times 10^{-3} - \rho_a \right) \left(\frac{1}{\rho_s} - \frac{1}{\rho_B} \right) \quad (8)$$

It is a requirement for manufacture that the actual density of standard weights be near enough to the assumed density of the apparent mass scale to which they are adjusted that the magnitude of (8) will always be small under normal conditions in laboratories near sea level.

The fact that there are two apparent mass scales widely used - one based on density 8.0 g/cm^3 and an older one based on 8.4 g/cm^3 - means that some caution is required on the part of the user. Conventional mass is generally preferred and reported for all calibrations where mass standards will be used to calibrate weighing instruments. For the most accurate work, the apparent mass scale should be abandoned in favor of substitution weighing with standards of known mass, density, and volume.

The user must decide the accuracy required of the particular mass measurement and choose a buoyancy correction technique commensurate with that accuracy.

The same considerations, which apply to the accuracy of buoyancy corrections in weighing by substitution, are easily extended to the other types of weighing indicated above.

There are many factors, which affect the accuracy of a mass measurement. The above has dealt only with those arising from problems of buoyancy.

Appendix B

Examples

Example 1:

The weight set of Table B-1 is used with an equal-arm balance to find the mass of a piece of single-crystal silicon. The following weights were used to balance the silicon: 10 g, 3 g. The balance pointer showed the silicon side to be light by 3.5 divisions. The 10 mg weight of Table 1 was used to find the sensitivity of the balance. When the weight was added, the pointer moved 10.3 divisions. At the time of the weighing, the following measurements were taken:

$$P = 748.1 \text{ mm Hg}$$

$$t = 22.3 \text{ }^{\circ}\text{C}$$

$$U = 37 \% \text{ relative humidity}$$

What is the mass of the silicon?

Answer:

From Eq. 1, or Table B-2, calculate

$$e_s = 1.3146 \times 10^9 \times e^{\left(\frac{-5315.56}{(22.3 + 273.15)}\right)} = 20.194 \text{ mm Hg}$$

$$\rho_a = \left\{ \frac{0.46460 [748.1 - (0.0037960)(37)(20.2)]}{(273.15 + 22.3)} \right\} 10^{-3}$$

$$\rho_a = 1.17194 \times 10^{-3} \text{ g/cm}^3$$

The density of silicon at 20 °C is 2.329 1 g/cm³ and its coefficient of linear expansion is 0.000 002 6 /°C.

Make use of Eq. 3c and Table B-1.

$$M_s = 10.0001264 + 3.0000459 = 13.000172 \text{ g}$$

$$V_s = (1.267 + 0.380) = 1.647 \text{ cm}^3$$

Calculate the sensitivity:

$$\text{Sensitivity} = \frac{0.010\,003\text{ g} - (0.003\,70)(1.172 \times 10^{-3})}{10.3} \text{ g/division}$$

$$= \frac{(0.009\,999)}{10.3} = 0.000\,971 \text{ g/division} = 0.971 \text{ mg/division}$$

$$M_x = \frac{[13.000\,172 - (1.647)(1.172 \times 10^{-3})] + [(0.971 \times 10^{-3})(-3.5)]}{\left(1 - \frac{1.172 \times 10^{-3}}{2.3291}\right)}$$

$$M_x = 13.001\,385 \text{ g}$$

Note that the thermal expansion is insignificant in this example.

Example 2:

Let us again consider the weighing performed in Example 1. This time, all we know about our weight set is that it has been adjusted to the 8.4 apparent mass scale at the best available commercial tolerance.

Using Eq. (4),

$$M_x = \left\{ \frac{\left[13.00 \left(1 - \frac{1.172 \times 10^{-3}}{8.390\,9} \right) \right] + \left[0.010 \left(1 - \frac{1.172 \times 10^{-3}}{8.390\,9} \right) \right] \left(\frac{-3.5}{10.3} \right)}{\left(1 - \frac{1.172 \times 10^{-3}}{2.3291} \right)} \right\}$$

$$M_x = 13.001\,329 \text{ g}$$

For routine weighing, it is sometimes satisfactory to assume that the temperature is 20 °C and the density of air is $1.2 \times 10^{-3} \text{ g/cm}^3$. Had this been done, the computed value for the silicon would be

$$M_x = \left\{ \frac{\left[13.00 \left(1 - \frac{1.2 \times 10^{-3}}{8.390\,9} \right) \right] + \left[0.010 \left(1 - \frac{1.2 \times 10^{-3}}{8.390\,9} \right) \right] \left(\frac{-3.5}{10.3} \right)}{\left(1 - \frac{1.2 \times 10^{-3}}{2.3291} \right)} \right\}$$

$$M_x = 13.001\,442\text{ g}$$

which is within 100 μg of the answer found in Example 1.

Example 3:

Another piece of silicon is measured on a single-pan microbalance. The balance weights were adjusted by the manufacturer to the conventional mass scale. The sensitivity of the balance has been determined to be exactly 1.000. This particular laboratory is well above sea level. At the time of the weighing, the following measurements were recorded:

$$P = 612.3\text{ mm Hg}$$

$$t = 23.4\text{ }^\circ\text{C}$$

$$U = 23\text{ \% relative humidity}$$

The balance reading was 15.00 g on the built-in weights and 0.000 358 g on the optical screen. What is the mass of the silicon?

Answer:

First, calculate e_s and ρ_a :

$$e_s = 21.59\text{ mm Hg}$$

$$\rho_a = 0.956 \times 10^{-3}\text{ g/cm}^3$$

Then, use Eq. 5:

$$M_x = \frac{(15.00 + 0.000\,358) \left(1 - \frac{0.956 \times 10^{-3}}{8.0} \right)}{\left(1 - \frac{0.956 \times 10^{-3}}{2.3291} \right)}$$

$$M_x = 15.004\,724\text{ g}$$

Example 4:

The built-in weights in Example 3 are actually stainless steel of density 7.78 g/cm^3 at 20 $^\circ\text{C}$. What is the approximate error caused by using the apparent mass scale?

Answer:

Using (7), the error is approximately

$$15 \text{ g} \left(0.2 \times 10^{-3} \text{ g/cm}^3 \right) \left(\frac{1}{7.78 \text{ g/cm}^3} - \frac{1}{8.0 \text{ g/cm}^3} \right) = 11 \mu\text{g}$$

This discrepancy, though larger than the precision of the best analytical balances, is actually well within the tolerance of ASTM Class 1 weights.

Table B-1. Example of calibration report data.

Mass (g)	Uncertainty (g)	Vol at 20 °C (cm ³)	Cubical Coefficient of Expansion (/°C)
100.000 941	0.000 025	12.674 39	0.000 045
50.000 463	0.000 016	6.337 19	0.000 045
30.000 293	0.000 014	3.802 32	0.000 045
20.000 158	0.000 011	2.534 87	0.000 045
10.000 130	0.000 013	1.267 44	0.000 045
5.000 042	0.000 0069	0.633 72	0.000 045
3.000 046	0.000 0046	0.380 23	0.000 045
2.000 006 3	0.000 003 3	0.253 49	0.000 045
1.000 014 4	0.000 003 0	0.126 74	0.000 045
0.499 953 8	0.000 001 6	0.030 12	0.000 020
0.299 961 4	0.000 001 2	0.018 07	0.000 020
0.199 949 84	0.000 000 87	0.012 05	0.000 020
0.099 963 78	0.000 000 91	0.006 02	0.000 020
0.049 986 59	0.000 000 72	0.003 01	0.000 020
0.029 991 00	0.000 000 77	0.001 81	0.000 020
0.020 005 70	0.000 000 66	0.007 41	0.000 069
0.010 002 77	0.000 000 86	0.003 70	0.000 069
0.004 997 06	0.000 000 70	0.001 85	0.000 069
0.003 002 99	0.000 000 76	0.001 11	0.000 069
0.002 001 97	0.000 000 66	0.000 74	0.000 069
0.001 000 83	0.000 000 86	0.000 37	0.000 069

Table B-2. e_s approximation in terms of temperature.

Temperature (°C)	e_s (mm Hg)
18	15.48
18.5	15.97
19	16.48
19.5	17
20	17.54
20.5	18.09
21	18.65
21.5	19.23
22	19.83
22.5	20.44
23	21.07
23.5	21.72
24	22.38
24.5	23.07
25	23.77
25.5	24.49
26	25.23
26.5	25.99
27	26.77
27.5	27.57
28	28.39
28.5	29.23
29	30.09
29.5	30.98

¹ Handbook 145 is out of print as of 2012. Section 7 was updated and published as a part of NISTIR 6969 in 2003, 2012, and 2014.